practice

How would you find:

- $$\int \frac{x-2}{x^2+x-6}\,\mathrm{d}x$$
- $$\int \frac{x^3}{x^2-1}\,\mathrm{d}x$$
- $$\int \frac{2\,x}{x^2+1}\,\mathrm{d}x$$

From now on, we're going to begin the class by practicing integration.

One key difficulty is figuring out which technique to use. Learning to identify a good strategy is one of the goals of this practice.

See § 7.5 and the review for Chp 7.

1. Factor, cancel the x-2.
   Ans = ln | x+3 | + C
2. Do long division, then partial fractions.
   Ans = (x^2 + ln | x^2 - 1 |)/2 + C
3. u substitution, u=x^2+1
   Ans = ln (x^2 + 1) + C

today:
homework 5 due (7.3.8, 7.3.22, 7.3.40, 7.4.20, 7.4.48, 7.4.50)
§ 4.4 - l'Hôpital's rule

wednesday:
mslc partial fractions workshop in CH 042 @ 12:30, 1:30, 3:30

thursday:
§ 7.8 - improper integrals
mslc l'Hôpital's rule workshop in CH 042 @ 12:30, 3:30

friday:
last drop day
webwork 5 due @ 11:55 pm
mslc webwork 5 workshop in SEL 040 @ 12:30, 1:30, 2:30, 3:30, 4:30

tuesday, 10 november:
§ 8.1 - arc length

thursday, 12 november:
§ 8.2 - surface area
quiz iv: § 4.4, 7.8
homework 6 due (4.4.28, 4.4.40, 4.4.58, 7.8.26, 7.8.36, 7.8.40)

monday, 16 november:
webwork extra credit ii due @ 6:00 am

midterm ii

Exams were generally good. The class average was 61%.

∞ is not a number

We must be careful doing arithmetic with ∞.

Suppose $f(x) \to A$ and suppose $g(x) \to B$ as $x \to x_0$ for some x_0; A and $B \neq 0$ numbers. Then

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \frac{A}{B}$$

The above is just a lazy short hand way of talking about limits.

The rule here: the limit of a quotient is the quotient of the limits, PROVIDED THEY EXIST.

Suppose instead $g(x) \to \infty$ with f as before. Then

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \frac{A}{\infty} = 0$$

But what if $f(x) \to \infty$? What is ∞/∞ ?

We have two rules that seem to be in conflict:

Anything divided by itself is one, anything divided by ∞ is 0.

Who wins?

examples

- $\lim_{x \to \infty} \frac{x}{x} = \lim_{x \to \infty} \frac{1}{x} = 0$
- $\lim_{x \to \infty} \frac{2\,x^2}{x^2} = \lim_{x \to \infty} 2 = 2$
- $\lim_{x \to \infty} \frac{x^3}{x^2} = \lim_{x \to \infty} x = \infty$

There is nothing special about 2; could just as easily get $-\pi$ or 7.

Moral: When dividing ∞/∞, can get anything.

indeterminate forms

When the following forms arise in a limit,

$$0/0 \qquad \infty/\infty$$

$$0 \cdot \infty \qquad \infty - \infty$$

$$0^0 \qquad 1^{\pm\infty}$$

$$\infty^0$$

they are called indeterminate forms.

There's nothing sacred about the sign of infinity.

0 times - ∞ is also an indeterminate form.

l'Hôpital's rule

Proof is in Appendix F.

Suppose f and g are differentiable and g'(x)≠0 near a except possibly at a. Suppose

$$\lim_{x\to a} f(x) = 0 \quad \text{and} \quad \lim_{x\to a} g(x) = 0$$

or

$$\lim_{x\to a} f(x) = \pm\infty \quad \text{and} \quad \lim_{x\to a} g(x) = \pm\infty$$

Then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

if the limit on the right exists.

l'Hôpital's rule, discovered (probably) by Johann Bernoulli says that if a 0/0 or ∞/∞ case arises, take the derivative of the top and bottom (separately).

We turn other indeterminate forms into one of these two cases.

example

evaluate

$$\lim_{x \to 0} \frac{\sin x}{x}$$

This is a 0/0 case, so we invoke l'Hopital and find the limit is 1.

example

evaluate

$$\lim_{x \to \infty} \frac{e^x + 1}{x^2 - 1}$$

This is an ∞/∞ case, so we invoke l'Hopital and find the limit is ∞.

example

evaluate

$$\lim_{x \to 0} \frac{x+1}{x-2}$$

L'Hôpital's rule does not apply here, since this is not an indeterminate form. The correct answer is 2, misuse of l'Hôpital would give 1.

example

evaluate

$$\lim_{x \to 0^+} x \ln x$$

We must turn this into a fraction. We do so by noting that multiplying by x is the same as dividing by 1/x. This gives -∞/∞, so we apply l'Hôpital and get 0.

We could also divide by 1/ln x, but that would lead a nastier derivative which wouldn't help us.

example

evaluate

$$\lim_{x\to\infty} x^2\, e^{-x}$$

Write multiply by e^{-x} as dividing by e^x, use l'Hôpital twice. The limit is 0.

example

evaluate

$$\lim_{x\to\infty} \left(\sqrt{x^2+x} - x\right)$$

This is Stewart 4.4.47.

Factor out an x from the square root, divide by 1/x instead, use l'Hôpital. Answer=1/2.

Alternatively, multiply by conjugate, simplify, no need for l'Hôpital.

example

evaluate

$$\lim_{x \to 0^+} (\tan(2x))^x$$

This is Stewart 4.4.52.

Find the limit of the ln of the expression. Let y=(tan(2x))^x, then work with ln y.

In evaluating, split up sin(2x)/(2x) and -x/cos(2x), take each limit separately.

The limit of y is e to the limit of ln y. Answer is 1.

coming soon

- read § 7.8
- webwork 5 due friday
- start homework 6 (due next thursday)
- start extra credit project 2,
  due 16 november @ 6:00 am